HW1

(1) Show that the advanced propagator defined by

$$D_{adv}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ikx}}{k^2 - m^2 - i\,\mathrm{sgn}(k_0)\varepsilon}$$

is non-zero only for $x_0 > 0$

$$\left.\begin{aligned} \mathrm{sgn}(x) &= +1 \quad x > 0 \\ &= -1 \quad x < 0 \end{aligned}\right\}$$

(2) Calculate the Euclidean propagator $D_E(x)$ averaged over space (ie final result will be a function only of Euclidean time)

(3) The partition function in the presence of an external source $J(x)$ is written

$$Z(J) = e^{\frac{1}{2}\int d^4x\, d^4y\, J(x)\, D(x-y)\, J(y)}$$

Evaluate this for a scalar field & source

$$J(x) = \theta(T-t)\,\theta(T+t)\left(q_1\delta^3(x) + q_2\delta^3(x-R)\right)$$

$R^\mu = (0, \vec{R}) \qquad T \gg R \gg 1/m.$

(3) continued

Following statistical mechanics we can compute a "free energy" as

$$E = -\frac{1}{\beta} \ln Z$$

Here $\beta = T$ a result from finite temperature field theory.

Hence calculate / estimate the energy as a function of $|R|$ & show that the force due to exchange of scalar particles is attractive.