HW 2

1) Let $\phi^c_n = \dfrac{\delta W(J)}{\delta J_n} = \langle \phi \rangle_J$ 　 n discrete label on lattice

if $\Gamma(\phi_c) = W(J) - \sum_n J_n \phi^c_n$.

a) Show that

$$\frac{\delta \Gamma}{\delta \phi^c_m} = - J_m .$$

b) Verify that

$$\sum_n \frac{\delta^2 W(J)}{\delta J_1 \delta J_n} \frac{\delta^2 \Gamma}{\delta \phi^c_n \delta \phi^c_2} = - \delta_{12}$$

c) Using the result

$$\frac{\partial}{\partial \alpha} M^{-1}(\alpha) = - M^{-1} \frac{\partial M}{\partial \alpha} M^{-1}$$

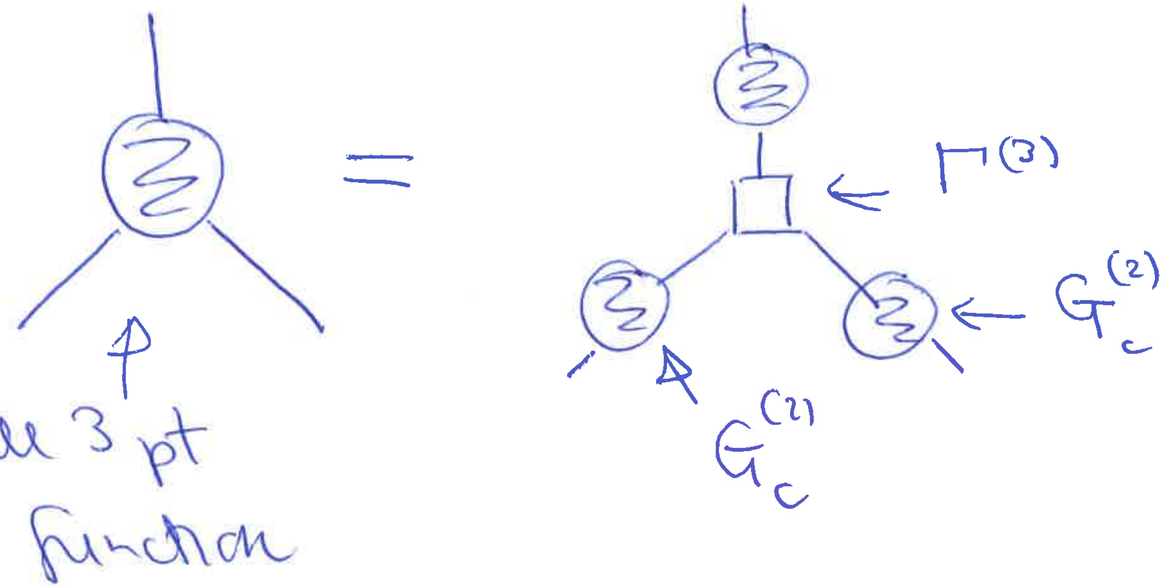
when $M_{\alpha\beta} = \dfrac{\delta^2 \Gamma}{\delta \phi_\alpha \delta \phi_\beta}$ & $M^{-1}_{\alpha\beta} = - \dfrac{\delta^2 W}{\delta J_\alpha \delta J_\beta}$

Show that

$$G^3_c(1,2,3) = \sum_{\alpha,\beta,\gamma} G^{(2)}_c(1,\alpha)\, G^{(2)}_c(2,\beta)\, G^{(2)}(3,\gamma)\, \Gamma^{(3)}_{\alpha\beta\gamma}$$

Interpretation: $\Gamma^{(3)}$ is the amputated 3 pt function (no external propagators)

i.e



(2) Counting dof & unitarity. Consider the Dirac equation (spin ½)

$$(i\not{\partial} + m)\psi = 0$$

Dirac spinor ψ has 4 components. But Wigner tells me I should just have 2 (spin states). To reconcile these statements go to p space $\Rightarrow$

$$(\not{p} + m)\psi(p) = 0$$

Now go to rest frame of particle $p = (m, 0, 0, 0)$.

a) Explain why EOM can be interpreted as a constraint on ψ that fixes up dof.

② continued

Now turn to spin 3/2. Introduce Rarita-Schwinger field ψ_μ^{α} $\alpha \equiv$ spinor index & $\mu \equiv$ vector index

The Lagrangian is given by

$$\mathcal{L} = \bar{\psi}_\mu \gamma^{\mu\nu\lambda} \partial_\nu \psi_\lambda + m \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu$$

where $\gamma^{\mu\nu\lambda} = \gamma^{[\mu}\gamma^{\nu}\gamma^{\lambda]}$ ← antisymmetrized in μ, ν, λ

$$\gamma^{\mu\nu} = (\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\frac{1}{2}.$$

a) Write down EOM for ψ

b) Act on EOM with ∂_μ to get a constraint

$$\partial_\mu \psi^\mu = (\gamma^\mu \partial_\mu)(\gamma^\nu \psi_\nu)$$

c) Act on EOM with γ_a to yield another constraint $\gamma^\mu \psi_\mu = 0$

d) How many dof are included in original ψ_μ^{α}? How many remain after 2 constraints are imposed?

e) Following the discussion of Dirac eqn (above) show ~~that~~ the EOM can be interpreted as a final constraint on ψ which produces correct # dof for spin 3/2