① $\delta\Gamma/\delta\varphi^c_m = \sum_n \delta W/\delta J_n \; \delta J_n/\delta\varphi^c_m$

$$- \sum_n \frac{\delta J_n}{\delta\varphi^c_m}\varphi^c_n - \sum J_n \delta\varphi^c_n/\delta\varphi^c_m$$

$$= -\delta_{nm} J_n = -J_m.$$

Since $\delta W/\delta J_n = \varphi^c_n$ & using chain rule.

$$\sum_n \frac{\delta^2 W}{\delta J_1 \delta J_n} \frac{\delta^2 \Gamma}{\delta\varphi^c_n \delta\varphi^c_2}$$

$$= \sum_n \frac{\delta\varphi^c_1}{\delta J_n} \cdot -\frac{\delta J_n}{\delta\varphi^c_2} = -\delta_{12} \text{ by chain rule}$$

$\delta\varphi^c_1/\delta\varphi^c_2 = \delta_{12}$

Interpretation: $\Gamma^{(2)} \equiv \frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}$

is inverse δJ $G^{(2)}_c = \frac{\delta^2 W}{\delta J \delta J}$. $\quad \Gamma^{(2)} = G^{(2)^{-1}}$

Using result

$$\frac{\delta}{\delta J_c} M^{-1}_{ab} = -M^{-1}_{ad} \frac{\delta M_{de}}{\delta J_c} M^{-1}_{eb}$$

when $M^{-1}_{ab} = \dfrac{\delta^2 W}{\delta J_a \delta J_b}$

thus $\dfrac{\delta^3 W}{\delta J_c \delta J_a \delta J_b} = - \dfrac{\delta^2 W}{\delta J_a \delta J_d} \dfrac{\delta M_{de}}{\delta J_c} \dfrac{\delta^2 W}{\delta J_e \delta J_b}$

ie $G_c^{(3)}(a,b,c) = - G^{(2)}(a,d) \dfrac{\delta M_{de}}{\delta J_c} G^{(2)}(e,b)$

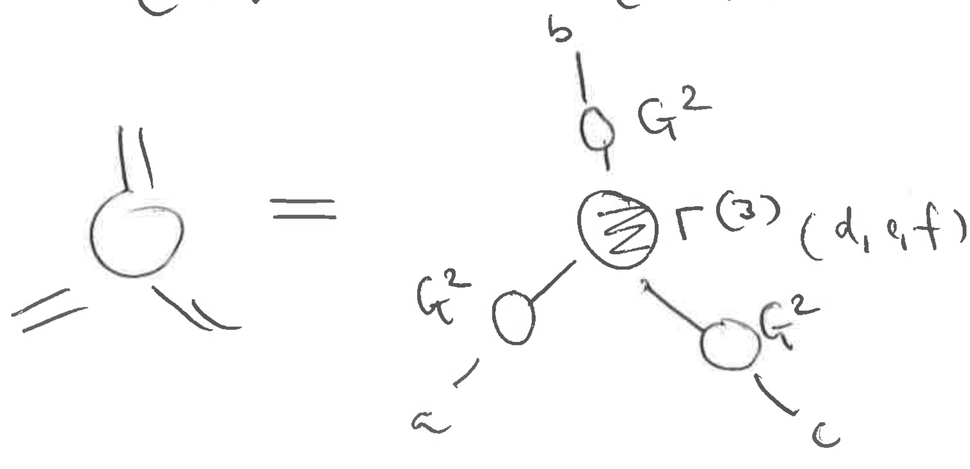
now $M_{de} = \Gamma^{(2)}(d,e) = \dfrac{\delta^2 \Gamma}{\delta \phi_d^c \delta \phi_e^c}$

$$G_c^{(3)}(a,b,c) = - G^{(2)}(a,d) \sum_f \frac{\delta^3 \Gamma}{\delta \phi_d^c \delta \phi_e^c \delta \phi_f^c} \frac{\delta \phi_f}{\delta J_c} G^{(2)}(e,b)$$

but $\dfrac{\delta \phi_f}{\delta J_c} = \dfrac{\delta^2 W}{\delta J_c \delta J_f}$ since $\phi_f = \delta W / \delta J_f$

$$\therefore\ G_c^{(3)}(a,b,c) = - G_c^{(2)}(a,d)\, \Gamma^{(3)}(d,e,f)\, G_c^{(2)}(c,f)\, G_c^{(2)}(e,b)$$

ie



(2) In rest frame:

$$(\not p + m)\psi = 0$$

$$\rightarrow \; 2m \cdot \underbrace{\tfrac{1}{2}(1+\gamma_0)}_{\text{projection operator } P_+}\psi = 0$$

i.e. $P_+^2 = P_+$

$P_+ P_- = 0$ when $P_- = \frac{1}{2}(1-\gamma_0)$

$\psi = P_+\psi + P_-\psi$ etc. etc.

removes <u>2 dof</u> ✓.

<u>Note</u>: can interpret Dirac equation as constraint boosted to arbitrary FOR!

<u>Rarita Schwinger eqn</u>

$$\mathcal{L} = \bar\psi_\mu \gamma^{\mu\nu\lambda}\partial_\nu\psi_\lambda + m\bar\psi_\mu\gamma^{\mu\nu}\psi_\nu$$

EOM $\Rightarrow$

$$\gamma^{\mu\nu\lambda}\partial_\nu\psi_\lambda + m\gamma^{\mu\nu}\psi_\nu = 0$$

op with $\partial_\mu \rightarrow$

$$\underbrace{\gamma^{\mu\nu\lambda}\partial_\mu\partial_\nu\psi_\lambda}_{=0} + m\gamma^{\mu\nu}\partial_\mu\psi_\nu = 0$$

$$\gamma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

$$\therefore \quad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$$

$$\therefore \quad (\gamma^\mu\gamma^\nu - \eta^{\mu\nu} I)\,\partial_\mu\psi_\nu = 0$$

ie $(\gamma^\mu\partial_\mu)(\gamma^\nu\psi_\nu) = \partial^\mu\psi_\mu$.

Op with $\gamma_\mu \Rightarrow$

$$\gamma_\mu\gamma^{\mu\nu\lambda}\partial_\nu\psi_\lambda + m\,\gamma_\mu\gamma^{\mu\nu}\psi_\nu = 0$$

this term

$\propto \gamma^{\nu\lambda}\partial_\nu\psi_\lambda$

must be built from 2 γ matrices antisymmetric in indices

$$= \gamma_\mu(\gamma^\mu\gamma^\nu - \eta^{\mu\nu})\psi_\nu$$

$$(4(\gamma^\nu) - \gamma^\nu)\psi_\nu$$

$$= 3\gamma^\nu\psi_\nu$$

$\therefore = 0$ by prev. constraint

$$\hookrightarrow \quad \gamma^\nu\psi_\nu = 0$$

ie 2 constraints on $\left.\begin{matrix}\gamma\cdot\psi = 0 \\ \partial\cdot\psi = 0\end{matrix}\right\}$

16 dof $\to$ 8 using these.

What about remaining 4 dof?

Return to <u>EOM</u>. & simplify using these

constraints

$$\gamma^{\mu\nu\lambda}\partial_\nu\psi_\lambda + m\underbrace{\gamma^{\mu\nu}}_{(\gamma^\mu\gamma^\nu - \eta^{\mu\nu})}\psi_\nu = 0$$

$$= -\psi^\mu.$$

$$\gamma^{\mu\nu\lambda} = \frac{1}{3}\left(\gamma^{\mu\nu}\gamma^\lambda + \gamma^{\lambda\mu}\gamma^\nu + \gamma^{\nu\lambda}\gamma^\mu\right)$$

since $\gamma\cdot\psi=0$

$$\therefore \gamma^{\mu\nu\lambda}\partial_\nu\psi_\lambda = \frac{1}{3}\Big(\cancel{\gamma^{\mu\nu}\gamma^\lambda}\partial_\nu\psi_\lambda$$

$$+ \gamma^{\lambda\mu}\gamma^\nu\partial_\nu\psi_\lambda$$

$$+ \gamma^{\nu\lambda}\gamma^\mu\partial_\nu\psi_\lambda\Big)$$

$(\gamma^\lambda\gamma^\mu - \gamma^\mu\gamma^\lambda)\gamma^\nu\partial_\nu\psi_\lambda$ $=0$ $\gamma\cdot\psi=0$ again

unless $\lambda=\nu$ when $\underline{\partial\cdot\psi=0}$

pull γ^λ to right $\Rightarrow \gamma\cdot\psi=0$

<u>unless</u> $\lambda=\mu$.

$\hookrightarrow (\gamma\cdot\partial)\psi_\mu$

Thus RS eq= collapses with these constraints to

$$(\gamma\cdot\partial - m)\psi^\mu = 0$$ like Dirac eq.

as for Dirac can interpret this equation in rest frame as a <u>projection</u> which removes $\frac{1}{2}$ component of ψ^μ

ie $8 \to 4$ dof as required for <u>spin $\frac{3}{2}$</u>