QFT II

Lecture 1

Assume that most / all of you have taken QFT I

(scalar field theory, QED, canonical quantization ...)

Start with quick review of scalar / fermion fields, path integrals & perturbation theory.

(maybe 2 weeks)

→ self-contained

Goals:

- non-abelian gauge symmetry & non-abelian fields
- quantization, renormalization
- BRST symmetry
- topological field configs eg instantons
- chiral gauge theories & anomalies

Maybe others eg SUSY, LGT, GUT

Why QFT?

- Relativity says that matter can be created out of energy
  so relativistic QM must ultimately be capable of describing all # particles
  (eg Dirac sea)
- Relativity demands space & time be treated on equal footing. But $\hat{x}$ operator, not t.
  Simplest option: downgrade $\hat{x} \to x$
- Origin of relativity is electromagnetism.
  In that case physical dof carried by field $A_\mu(x,y,z,t)$ $\mu = 1..4$
  Furthermore, decompose A_μ into Fourier modes each of which is interpreted as particle – the photon
  – Use field idea for all elementary particles
- Bonus: allows us to think of non-trivial ground states when field is not zero ....

Scalar field

Spin zero particles described by scalar field ϕ (eg pions, Higgs...)

Classical dynamics obtained by variation of action $S[\phi] \equiv S[\phi(x_\mu)]$

$$S = \int d^4x \, \mathcal{L}(\phi, \partial\phi)$$

Principle of least action

EOM corresponds to $\delta S = 0$

$$\delta S = \int d^4x \left(\frac{\delta \mathcal{L}}{\delta \phi}\delta\phi + \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \delta(\partial_\mu \phi) \right)$$

simplify ...

$$\partial_\mu \left(\frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \delta\phi \right) = \partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \delta\phi + \frac{\delta \mathcal{L}}{\delta (\partial_\mu \phi)} \partial_\mu \delta\phi$$

$$\therefore \delta S = \int d^4x \left(\frac{\delta \mathcal{L}}{\delta \phi} - \partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \right) \delta\phi$$

$$+ \underline{\int d^4x \, \partial_\mu \left(\frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \delta\phi \right)} \; *$$

EOM assumes $\delta\phi = 0$ on boundary

Euler-Lagrange eqn $\Rightarrow$

$$\frac{\delta \mathcal{L}}{\delta \phi} - \partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} = 0$$

Note: Suppose you have symmetry of system

this means $\delta_\epsilon S = 0$ for some specific δ_ϵ

ϵ is small parameter

* can be written $\int d^4x \, \partial_\mu J^\mu_\epsilon$

conserved current associated with symmetry (later ...)

$\mathcal{L}$ (scalar field) $\leftarrow (\partial_\mu \phi)(\partial^\mu \phi) = \left(\frac{\partial \phi}{\partial t}\right)^2 - (\nabla\phi)^2$

$$= \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4$$

EL $\rightarrow$ $\quad -\partial_\mu(\partial_\mu \phi) - m^2\phi - \frac{\lambda}{3!}\phi^3$

$\lambda = 0$ Klein-Gordon eqn
(note symmetry x, t derivs)

Lorentz inv.

$(\partial_\mu \phi)(\partial_\mu \phi) \sim -\phi \Box \phi \qquad \Box$ L.I

$\phi(x') = \phi(x)$ defn of scalar field

invariant under L.T

Quantization ?

Follow Feynman. Amplitude for particle to propagate between 2 points is sum over all paths between these points

(think 2 slit experiment ...)

$$a \sim \sum_{\text{paths}} e^{iS/\hbar} \qquad S = \text{action along path}$$

Field Theory assume similar

$$\langle \phi_f | \phi_i \rangle = N \int_{\substack{\phi=\phi_i \; t\to-\infty \\ \phi=\phi_f \; t\to+\infty}} D\phi \, e^{iS[\phi]/\hbar}$$

In practice all physics can be derived from vacuum to vacuum amplitude

(in presence of source – later)

$$Z = \int D\phi \, e^{iS/\hbar}$$

partition function or vacuum generating functional

Things to note

1. $\hbar \to 0$ configurations of ϕ where $\delta S = 0$ dominate integrand

i.e. classical limit automatically recovered.

2. All properties of quantum theory can be deduced from knowledge of its Green functions (correlation functions)

$$\langle \phi(x_1) \cdots \phi(x_N) \rangle = \frac{1}{Z} \int D\phi \, \phi(x_1) \cdots \phi(x_N) e^{iS}$$

to get them add source term

$$\int d^4x \, J(x) \phi(x) \text{ to } S$$

$$Z \to Z[J] = \int D\phi \, e^{i[S + \int J\phi]}$$

(note using natural units where $\hbar = 1$ now on)

Differentiate $Z(J)$ wrt J to generate Green functions

Euclidean space

These (functional) integrals poorly define (oscillate)

Better to define them in Euclidean space

$t \to -i\tau$.

$iS \to -S_E$

with $S_E = \left((\partial_\mu \phi)^2 + \frac{m^2 \phi^2}{2} + \frac{\lambda \phi^4}{4!} - J\phi \right)$

$$e^{iS} \to e^{-S_E/\hbar}$$

(damped).

Measure

How to define functional integral measure $D\phi$?

Replace cont: spacetime by grid or lattice

$x \to x_i = ia \quad i = 1 \ldots N$.

$\phi(x_i) \equiv \phi_i$ etc. $\partial_i \phi \sim \frac{\phi_i - \phi_{i+1}}{a}$

$$D\phi \equiv \prod_{i=1}^{N} d\phi_i$$

take limit $N \to \infty$ $(a \to 0)$ at end ... subtle

(renormalization)