hw4

(1) The β function for an $SU(N)$ gauge theory coupled to N_f flavors of fermion in the fundamental representation of the group is given to 2 loops by the expression

$$\frac{d\alpha}{d\ln\mu} = -2\alpha\left(b_1\frac{\alpha}{4\pi} + b_2\left(\frac{\alpha}{4\pi}\right)^2 + \cdots\right)$$

where $\alpha = g^2/(4\pi)^2$ and

$$b_1 = \frac{1}{3}(11C_A - 2N_f) \quad \text{1 loop}$$

$$b_2 = \frac{1}{3}(34C_A^2 - 2(5C_A + 3C_f)N_f) \quad \text{2 loops}$$

where C_A = quadratic Casimir for adj rep

$= N$ for $SU(N)$

C_f = quadratic Casimir for fundamental rep

$= \frac{N^2-1}{2N}$ $SU(2)$

a) Derive an expression for $\alpha(\mu)$ for the 1-loop β function (assume $\alpha = \alpha_0$ at $\mu = \mu_0$)
Show that the theory is asymptotically free if $N_f < 11 C_A/2$
Does this result change at 2 loops?

b) Consider the case $N=2$. Sketch the β function for $N_f = 0$ and $N_f > 11$
For what values of N_f does the β function develop a zero away from the origin?

c) What is the value of α when this new zero occurs?
Can I trust this calculation?

d) If the β function develops such a new zero describe the theory as $\mu \to 0$

(2) Consider the BRST transformations:

$$Q A_\mu^a = D_\mu^{ab} c^b$$

$$Q c^a = -\tfrac{1}{2} g f^{abc} c^b c^c$$

where A_μ is the gauge field & c the ghost

Show that

$$Q^2 A_\mu^a = 0$$

(3) Compute the anomalous dimension γ_A for A_μ in pure YM theory using the results for Z_i (in $\overline{MS}$ scheme) derived in class.

Write down a differential equation (RG equation) governing the gluon propagator for SU(2) in terms of $\beta(g)$ & γ_A