QED

want

$$Z(\bar{\eta},\eta,J) = \int DA\,D\psi\,D\bar{\psi}\; e^{iS(\bar{\eta},\eta,J)}$$

$$S = \int d^4x\,\mathcal{L} + \int d^4x\; \bar{\eta}\psi + \bar{\psi}\eta + J\cdot A$$

$\uparrow J^\mu A_\mu$

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi - m\bar{\psi}\psi$$

$$+ \frac{1}{\alpha}(\partial\cdot A)^2$$

covariant derv needed for gauge invariance

$$\not{D} = \gamma^\mu(\partial_\mu + ieA_\mu)$$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 \Leftarrow e\bar{\psi}\gamma^\mu A_\mu\psi$$

note: is of $A\cdot J$ type...

$$Z = \int DA\,D\psi\,D\bar{\psi}\; e^{iS_0}\left(1 + \int \mathcal{L}_1 + \frac{1}{2!}\left(\int \mathcal{L}\right)^2 + \cdots\right)$$

$e = 0.3$ natural units

so higher terms suppressed

As for scalars →

$$Z(\eta, \bar{\eta}, J) = e^{i\int d^4x\, \mathcal{L}_1\left(\frac{\delta}{\delta J}, \frac{\delta}{\delta \eta}, \frac{\delta}{\delta \bar{\eta}}\right)}$$

$$\times Z_0(\eta, \bar{\eta}, J)$$

where

$$\mathcal{L}_1 = \frac{1}{i}\frac{\delta}{\delta J}\; \frac{i\,\delta}{\delta \eta_\alpha(x)}\gamma^\mu{}_{\alpha\beta}\frac{\delta}{\delta \bar{\eta}_\beta(x)}$$

&

$$Z_0 = e^{\frac{i}{2}\int d^4x\, d^4y\, J^\mu(x)\Delta_{\mu\nu}(x-y)J^\nu(x)}$$

$$\times\, e^{i\int d^4x\, d^4y\, \bar{\eta}(x) S(x-y)\eta(y)}$$

with S as above &

$\Delta_{\mu\nu}$ (photon propagator)

$$= \int \frac{d^4k}{(2\pi)^4}\frac{g_{\mu\nu}}{k^2 - i\epsilon}$$

in Feynman gauge $\underline{\alpha = 1}$

$\boxed{\text{Greens functions given by derivatives of } \ln Z(\eta, \bar{\eta}, J) \text{ wrt sources}}$

As for scalars terms that appear in
this perturbation expansion in e
can be put into 1:1 correspondence with

Feynmann diagrams

Can be given a physical interpretation
as illustrating creation / propagation &
annihilation of elementary excitations of
the fields

Propagator lines give amplitude to
propagate while particles interact at
vertices which join together such
propagators ...

In detail for QED ...

① Draw all topologically distinct graphs with correct # external lines (+type) (i.e. n for $G_c^{(n)}$ ...)

② Assign (in k space) propagators $S(p)$, $\Delta(p)$ to all such lines

③ Assign $ie\gamma_\mu$ to each vertex where lines meet.

④ Conserve (4) momenta at each vertex

⑤ Integrate $\int \frac{d^4p}{(2\pi)^4}$ over all internal momenta on non-external lines

$\Uparrow$

this is all similar to scalar field theory

In addition for spin 1/2, spin 1 additional factors

(6) Attach spinor wavefunctions $u(p), v(p)$ to external fermion lines to soak up spinor indices

(7) Similarly external photon lines carry polarization vectors $\epsilon_\mu(p)$

(8) Multiply each closed fermion loop by minus one. (Pauli)

(9) Move along lines contracting all indices ...
(10) symmetry factors!

to calculate scattering crosssections one needs to pay attention to (6) & (7) (see textbook)

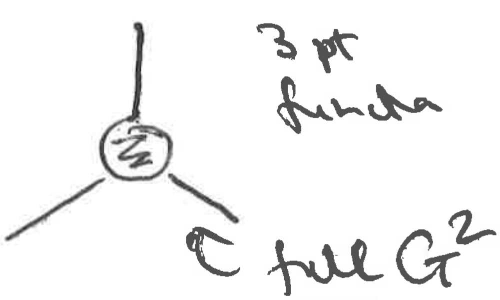
In addition it is useful to remember various formulae ~~that~~ arise when you do traces over γ matrices, sums of spin states & polarization vectors -- not here

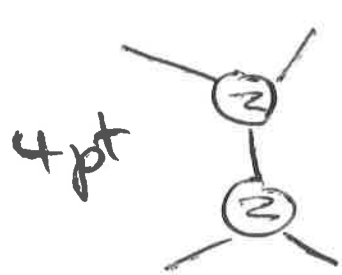
Instead, here I want to draw on amputated Feynman graphs

— remove all external propagators

In fact if I restrict myself to amputated diagrams which cannot be cut into 2 pieces ~~without~~ by cutting single internal line I will just end up with the proper vertices I discussed before in scalar field theory

Any Feynman graph (connected) can be built by connecting such vertices together with full propagators but no loops





skeleton graphs

Indeed we saw that entire quantum effective action could be built from such quantities

We will thus just concentrate on them. All the subtleties of renormalization will be apparent when we try to compute them.

Indeed once we have renormalized the 1PI proper vertices we will have a completely finite quantum theory

In practice in this class we will only compute the 1 loop diagrams

this is already some work but will be enough to illustrate most of the important physics...

We saw that the quantum action contained an ∞ # of proper vertices or interactions that can arise via quantum effects

Even when we restrict to terms that are Lorentz invariant, P, C, T invariant gauge invariant there are still a lot!

eg $\mathcal{O} = \int \bar{\psi} \gamma_\mu \gamma_\nu D^\mu D^\nu \psi$

However there is a well-defined sense in which these interactions can be ignored at low energies or when the physical cutoff is removed. In the old days one would have said they were non-renormalizable ops

Now with Wilson we say they are irrelevant

Simple dimensional analysis shows us why

Examine kinetic term for fermions

$$\int \bar{\psi} \not{\partial} \psi \, d^4x$$

since S is dimensionless

$$[\psi] \sim a^{-3/2} \qquad a \sim \text{length}$$
(lattice spacing)

Similarly $F_{\mu\nu}^2 \to [A_\mu] = a^{-1}$

thus $[O]$ (above) $= a^4 (a^{-3/2})^2 (a^{-1})^2$
$$= a^{-1}$$

this means its coupling $g_0 \sim a$

So naively as $a \to 0$ these operators go away!

– they are irrelevant

So in practice can restrict ourselves to ops with mass dimension 4 or less

$\boxed{\text{For QED these are already in } \mathcal{L}\text{!}}$

Thus the most general Lagrangian we need to consider even after quantum correction is

$$\mathcal{L} = \underset{\underset{\underline{\underline{\text{free}}}}{\uparrow}}{\mathcal{L}_0} + \mathcal{L}_1$$

$$\mathcal{L}_1 = Z_1 e \bar{\psi} \not{A} \psi + i(Z_2 - 1)\bar{\psi} \not{\partial} \psi$$
$$- (Z_m - 1) m \bar{\psi}\psi$$
$$- \tfrac{1}{4}(Z_3 - 1) F_{\mu\nu}^2$$

where we know $Z_i \to 1$ as $e \to 0$

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to find these Z's need to calculate 1PI, amputated Feynman graphs

At one loop

remove photon propagators

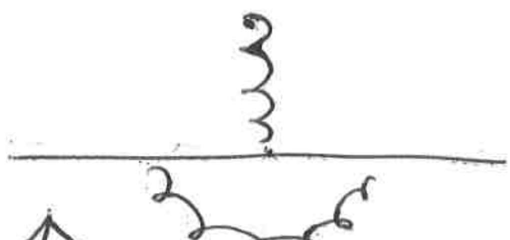
called vacuum polarization Π

$\hookrightarrow Z_3$

remove fermion propagators

called self energy

$\Sigma \to (Z_2, Z_m)$



called the vertex diagram.

remove external propagators