hw4-solutions

(1) a) $$\frac{d\alpha}{d\ln\mu} = -2\alpha\left(\frac{11C_A - 2N_f}{3}\right)\frac{\alpha}{4\pi}$$

<u>where</u> $\alpha = g^2/(4\pi)$ ← note <u>typo</u> in question

$$\rightarrow \int_{\alpha_0}^{\alpha}\frac{d\alpha}{\alpha^2} = -\int_{\mu_0}^{\mu}\frac{b_1}{2\pi}\,d\ln\mu \qquad b_1 = \frac{11C_A - 2N_f}{3}$$

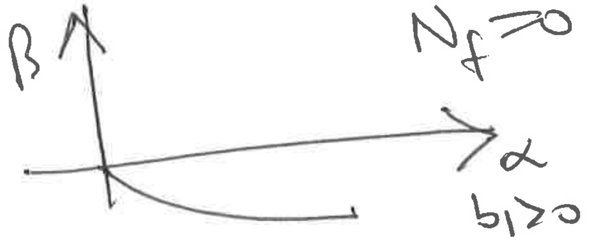
$$\hookrightarrow \alpha = \frac{\alpha_0}{1 + \frac{b_1\alpha_0}{2\pi}\ln\mu/\mu_0}$$

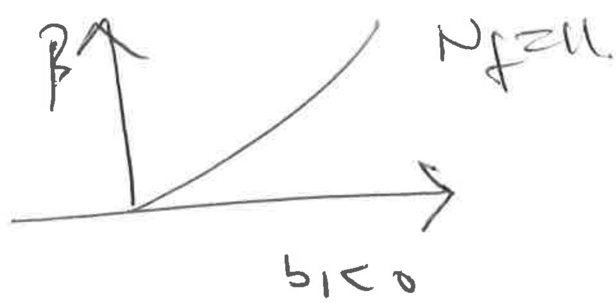
if $N_f < 11C_A/2$ $\quad \beta = d\alpha/d\ln\mu < 0$

→ asymptotic freedom

examination of b_2 shows that this requirement is sufficient to keep $\beta<0$ at 2 <u>loops</u>

b) $N=2 \leftarrow C_A = 2$





$\beta(g)$ develops a zero away from origin ②

if $b_1 > 0$ & $b_2 < 0$

For $N=2$ $C_A = 2$, $C_F = 3/4$.

$b_2 < 0 \Rightarrow \quad \underline{5 < N_f < 11}$

c) $\alpha_c = -\dfrac{b_1}{b_2}$ (set $\beta = 0$)

$= \dfrac{22 - 2N_F}{136 - 49/2 N_F}$

put $N_F = 6 \longrightarrow \alpha_c = +0.91$

$\underline{N_F = 10} \longrightarrow \alpha_c = 2/109 = 0.018$.

Thus $N_F = 10$ calc produce an IR fixed pt at weak coupling. Probably reliable but $N_F = 6$ theory gives $\alpha = 0.9$ ← 2 loop not reliable...

$\mu \to 0$ $\alpha \to \alpha_c$ ($\beta = 0$)

ie lower boundary of conformal window needs non-perturbative determination...

(2) (B)

$QA_\mu^a = D_\mu^{ab} c^b$

$Qc^a = -\tfrac{1}{2} g f^{abc} c^b c^c \Leftarrow$ show $Q^2 A_\mu^a = 0$

Quick soln : (or use Srednicki page 149)

$$Q^2 A_\mu = Q(D_\mu c)$$

$$= D_\mu Qc + (QD_\mu)c$$

but $D_\mu = \partial_\mu + gfA_\mu$

$$\therefore Q^2 A_\mu = D_\mu Qc + gfQA_\mu c$$

$$= D_\mu Qc + gf D_\mu c\, c$$

$$= D_\mu Qc + \frac{g}{2} f D_\mu (cc)$$

$$= D_\mu \left(Qc + \frac{g}{2} f cc\right)$$

$$= 0$$

(B) $\qquad$ (C)

$$\gamma_A = \frac{1}{2}\frac{\partial \ln Z_3}{\partial \ln \mu}$$

$$Z_3\ (\overline{MS}) = 1 + \left(\tfrac{5}{3}T(A) - \tfrac{4}{3}n_F T(R)\right)\frac{g^2}{8\pi^2}\frac{1}{\epsilon}$$

now/

$$\gamma_A = \frac{1}{2}\frac{\partial \ln Z_3}{\partial g}\frac{\partial g}{\partial \ln \mu}$$

to $O(\epsilon)$ $\quad \dfrac{\partial g}{\partial \ln \mu} = -\dfrac{\epsilon g}{2}$

$$\therefore\ \gamma_A = \frac{1}{2}\left(\tfrac{5}{3}T(A) - \tfrac{4}{3}T(R)n_F\right)\frac{2g}{8\pi^2 \cancel{\epsilon}}\ \frac{-\cancel{\epsilon}g}{2}$$

$$\gamma_A = -g^2/16\pi^2\left(\tfrac{5}{3}T(A) - \tfrac{4}{3}n_F T(R)\right)$$

Callan Symanzik eq^n:

$$\left(\frac{\partial}{\partial \ln \mu} + \beta\,\partial/\partial g + 2\gamma_A\right)\Delta(k^2) = 0$$
