hw3 - solutions

$$S(\not p)^{-1} = \not p + m - \Sigma(\not p)$$

on shell renormalization scheme

if $m = m_{\text{physical}}$ = pole of S requir

① $\Sigma(-m) = 0$

② $\Sigma'(-m) = 0 \leftarrow$ residue of pole $= 1$.
(fixes Z_1)

ie $\lim_{\not p \to -m} (\not p + m) S(\not p) = 1$

$\Rightarrow \Sigma / \not p + m = 0$ i $\Sigma'(-m) = 0$

the requirement that $\Sigma(-m) = 0$ ensures that

$$\Sigma(\not p) = \frac{e^2}{8\pi^2} \int_0^1 dx \left[((1-x)\not p + 2m) \ln D/D_0 + k_2(\not p + m) \right]$$

wher $D_0 = x^2 m^2 + (1-x) m_\gamma^2$ $\quad (D(p^2 = -m^2) = D_0)$

$$\frac{\partial \Sigma}{\partial \not p} = \frac{e^2}{8\pi^2} \left(\int_0^1 dx \left[(1-x) \ln D/D_0 + ((1-x)\not p + 2m) \frac{1}{D} \frac{\partial D}{\partial \not p} \right] \right.$$

$$\frac{\partial D}{\partial \not p} = 2x(1-x)\not p$$

At $\not p = -m$ must have: (2)

$$0 = \frac{e^2}{8\pi^2}\left(\left[-\int_0^1 dx \, \frac{(-(1-x)m + 2m)\, 2x(1-x)m}{D_0}\right] + k_2\right)$$

ie

$$k_2 = + \int_0^1 dx \, \frac{2x(1-x^2)m^2}{x^2 m^2 + (1-x)m_\gamma^2}$$

let $\epsilon = (m_\gamma / m)^2$

ie $$k_2 = \cancel{+} 2 \int_0^1 dx \, \frac{x(1-x^2)}{x^2 + (1-x)\epsilon}$$

Integrand behaves as $\frac{dx}{x}$ for ϵ small & close to $x=0$

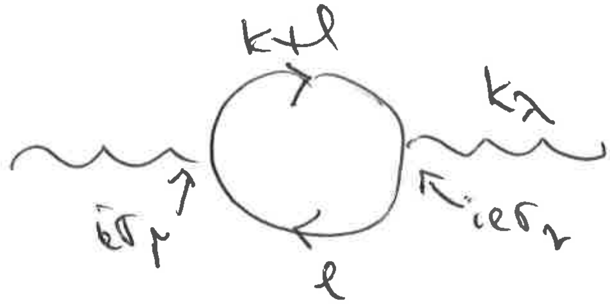
neglecting ϵ term in denominator — cutoff at $x \sim \sqrt{\epsilon}$

$$k_2 \simeq -2 \int_{\sqrt{\epsilon}}^1 \left(\frac{1}{x} - x\right) dx$$

$$= + 2\ln\sqrt{\epsilon} + 2 \cdot \frac{1}{2} x^2 \Big|_{\sqrt{\epsilon}}^1$$

$$= 1 - 2\ln m/m_\gamma$$

(3)



$$i\Pi_{\mu\nu} \sim (-1)e^2 \int \frac{d^3 l}{(2\pi)^3} \frac{\mathrm{Tr}\left(\sigma_\mu(-\not{k}-\not{l}+m)\sigma_\nu(-\not{l}+m)\right)}{(l^2+m^2)((k+l)^2+m^2)}$$

want term linear in k

$$\Rightarrow \left.\frac{\partial \Pi_{\mu\nu}}{\partial k_\lambda}\right|_{k=0}$$

leading term comes from differentiating numerator

terms linear in $l \to 0$

$$-e^2 \int \frac{d^3 l}{(2\pi)^3} \frac{\mathrm{Tr}\left(\sigma_\mu (-\sigma_\lambda) \sigma_\nu (\cancel{\ } + m)\right)}{(l^2+m^2)(l^2+m^2)^2}$$

at $k=0$

but $\mathrm{Tr}(\sigma_\mu\sigma_\lambda\sigma_\nu) = 2i\epsilon^{\mu\lambda\nu}$

$$\cong -ie^2 m\, \epsilon^{\mu\nu\lambda} \int \frac{d^3 l}{(2\pi)^3} \frac{1}{(l^2+m^2)^2}$$

$$\sim -ie^2 \epsilon^{\mu\nu\lambda} \times \underset{\text{finite}}{\#}$$

$\Leftarrow$ U.V convergent!

$$\sim \int \frac{1}{m} \frac{q^2 dq}{(q^2+1)^2}$$

Actually result of integral clearly only ④
depends on m^2

$\therefore$ amplitude $\sim - \# i e^2 \epsilon^{\mu\nu\lambda} \frac{m}{|m|}$

$\therefore$ quantum effective action contains

$$\int i e^2 \frac{m}{|m|} A_\mu \partial_\nu A_\lambda \epsilon^{\mu\nu\lambda} \times \#$$

<u>Chern-Simons action</u> !

<u>topological</u> (doesn't need metric to contract indices)

<u>gauge invariant</u> (up to boundary terms)

<u>Notice</u>: induced even as <u>$m \to 0$</u>

"parity anomaly"

ie $\int \partial_\mu \alpha \partial_\nu A_\lambda \epsilon^{\mu\nu\lambda} \sim \left[\oint \alpha \partial_\nu A_\lambda \epsilon^{\mu\nu\lambda} \right]_{\text{boundary}}$

CS breaks parity: fermion mass (3D) breaks parity
effect of fermion mass $\to$ CS <u>even</u> after $m \to 0$