hw5

(1) For the kink solution discussed in class

$$\varphi = v \tanh \frac{m}{2}(x - x_0)$$

Show that the Lorentz boosted solution is also a solution of the EOM.

$$\varphi(x,t) = v \tanh\left[\frac{1}{2}\gamma m\,(x - x_0 - \beta t)\right]$$

with

$$\gamma = (1-\beta^2)^{-1/2}$$

Show that the energy of this solution is

$$E = \gamma M = (p^2 + M^2)^{1/2}$$ where M is the energy / mass of the kink at rest.

& $p = \gamma \beta M$ is its momentum. found by integrating the momentum density

$$T^{01} = \frac{\partial \varphi}{\partial x}\frac{\partial \varphi}{\partial t}$$

(2) Show that the expression

$$\int d^3x\, \epsilon^{ijk}\, \mathrm{Tr}\left((g\partial_i g^\dagger)(g\partial_j g^\dagger)(g\partial_k g^\dagger)\right)$$

is invariant under (infinitessimal) smooth deformations $g \to g + \delta g$

Argue ~~that~~ it is also independent of coordinate choice on 3d space ie it is a topological invariant

(3) In general we can interpret the instanton as a tunneling solution that interpolates between a Yang-Mills vacuum with winding # n at $x_4 = -T$ & another with winding # m. at $x_4 = +T$, $T \to \infty$

with $\langle n|H|m\rangle \sim e^{-S_{instanton}} \sim e^{-|m-n|S_0}$

Hamiltonian

General arguments show that this matrix element only depends on $(n-m)$

a) Show that the eigenstates of H are "θ vacua" of the form

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{in\theta} |n\rangle$$

(Hint: show that $\langle\theta'|H|\theta\rangle$ diagonal)

b) Show that if the only non-zero matrix elements of $\langle m|H|n\rangle$ are $m = n \pm 1$ that the energy is proportional to $-\cos\theta$