Gauge Anomalies

Seen that massless Dirac Lagrangian separates into 2 indep pieces

$$\mathcal{L} = \bar{\psi}_+ i\not{\partial} \psi_+ + \bar{\psi}_- i\not{\partial}\psi_-$$

$$= \psi_R^\dagger i\sigma\cdot\partial\psi_R + \psi_L^\dagger i\bar{\sigma}\cdot\partial\psi_L$$

Therefore it is possible to gauge L & R pieces individually

$$\left\{ \begin{array}{l} \sigma = (\sigma_i, I) \\ \bar{\sigma} = (\sigma_i, -I) \\ \psi_R = \psi_+ = \frac{1}{2}(1+\gamma_5)\psi \\ \text{etc} \end{array} \right.$$

↳ $\boxed{\text{chiral gauge theory}}$

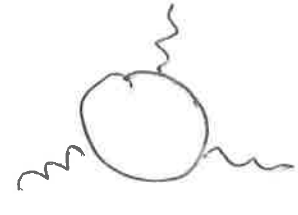
In Dirac language this will yield $\gamma_\mu\gamma_5$ axial couplings to gauge bosons.

$\longrightarrow$ possibility of anomalies

However now an anomaly would imply that the corresponding gauge current is not conserved

$\longrightarrow$ breakdown of gauge invariance, unitarity & renormalisability.

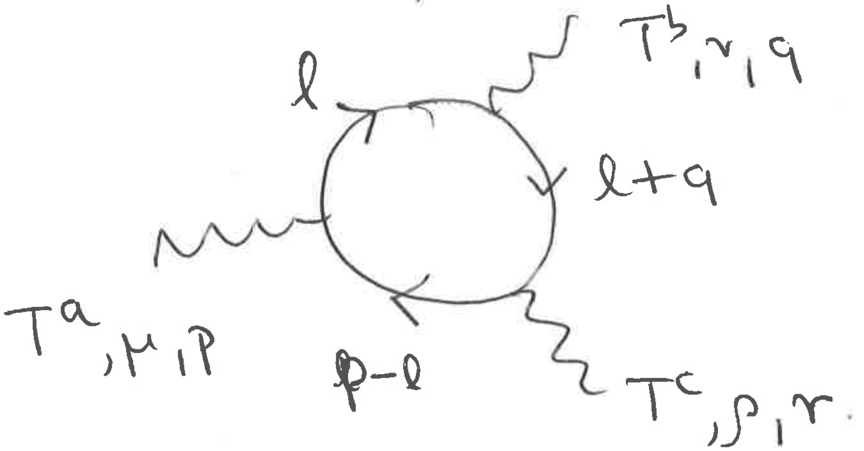
Consider 1 loop diagram with 3 external bosons



$$\mathcal{L} = i\bar{\psi}\gamma^\mu D_\mu P_+ \psi - \tfrac{1}{4}F_{\mu\nu}^2$$

$$P_+ = \tfrac{1}{2}(1+\gamma_5)$$

write using Dirac fermions $\rightarrow$ propagator: $\dfrac{P_+ \not{p}}{p^2}$



amplitude $\sim (-1)\,\mathrm{Tr}(T^a[T^b, T^c]_+)$

$$\times \int \frac{d^4 l}{(2\pi)^4} \frac{N^{\mu\nu\rho}}{(l-p)^2 l^2 (l+q)^2}$$

only symmetric piece contributes - other yields f^{abc} $\Leftarrow$ absorbed into 3 pt vertex

when

$$N^{\mu\nu\rho} = \mathrm{Tr}\left[(-\not{l}+\not{p})\gamma^\mu(-\not{l})\gamma^\nu(-\not{l}-\not{q})\gamma^\rho P_+\right]$$

note P_+ commutes with 2 γ's & $P_+^2 = P_+$

only single P_+ remains in whole trace...

In fact it is only the insertion γ_5 that is problematic ...

Indeed one sees that the diagram is <u>same</u> as the one we considered earlier for theories with just global symmetries

$\longrightarrow$ thus expect $\partial^\mu J_{\mu 5} \neq 0$ q. theory

furthermore

$$\mathrm{Tr}(T^a [T^b, T^c]_+) = A(R)\, d_{abc}$$

anomaly coeff

$$\partial_\mu J^{\mu 5}_L = -\frac{1}{2} A(R) \frac{g^2}{8\pi^2} \mathrm{Tr}\, F_{\mu\nu} \tilde{F}_{\mu\nu}\, d_{abc}$$

while

$$\partial_\mu J^{\mu 5}_R = +\frac{1}{2} A(R) \frac{g^2}{8\pi^2} F\tilde{F}\, d_{abc}$$

thus

$$\partial_\mu J^\mu = \partial_\mu (J^{\mu 5}_L + J^{\mu 5}_R) = 0 \leftarrow \text{vector current ok}$$

$$\partial_\mu J^{\mu 5} = \partial_\mu (J^{\mu 5}_L - J^{\mu 5}_R) \neq 0 \leftarrow \underline{\text{chiral}} \text{ current}$$

Thus in a chiral gauge theory we must be careful to arrange for these anomalies to cancel

$\hookrightarrow$ places severe constraints on possible chiral gauge theories

(SM is one example: here anomaly cancels — see later)

Simplest example

$$S = \int i\bar{\psi}_+ \gamma\cdot D \psi_+ - \tfrac{1}{4} F_{\mu\nu}^2$$

massless by gauge invariance (no ψ_- state to make fermion bilinear)

assume ψ_+ in some rep of gauge group (eg $SU(N)$)

This effect is disasterous → ruins G.I etc.

It is a 1 loop effect (no additional issues at higher loops)

The only known way out is if

$$A(R) = 0 \leftarrow \text{real / pseudoreal reps}$$

<u>or</u>

eg adjoint, SO(N) groups, fund of SU(2) . .

set of chiral fermions running around loop generate a set of $A(R)$'s ~~that~~ <u>cancel</u>.

↑

Simple way - Dirac fermions since L & R components cancel.

or more generally

$$\sum_{\text{reps } R} \mathrm{Tr}\left(T^a_R [T^b_R, T^c_R]_+\right) = 0$$

this is what happens in SM!

$U(1)_Y^3$: $(2Y_L^3 - Y_e^3 - Y_\nu^3)$

$$+3(2Y_Q^3 - Y_u^3 - Y_d^3) = 0$$

$SU(2)^2 \times U(1)_Y$:

$$2Y_L + 6Y_Q = 0$$

← lefthanded leptons \ quarks

$SU(3)^2 \times U(1)$:

$$6Y_Q - 3Y_u - 3Y_d = 0$$

$Y_L = -\frac{1}{2}$ $\quad Y_e = -1$ $\quad Y_Q = \frac{1}{6}$ $\quad$ SM.

$Y_u = 2/3$ $\quad Y_d = -1/3$ $\quad Y_\nu = 0$

es for $U(1)^3$:

$$\partial_\mu J^{\mu Y} = \Big(\sum_{\text{left}} Y_l^3 - \sum_{\text{right}} Y_r^3\Big) \frac{g'^2}{32\pi^2} \epsilon^{\mu\nu\alpha\beta} B_{\mu\nu} B_{\alpha\beta}$$

① No $SU(2)^3$ anomalies ← pseudoreal

② No $SU(3)^3$ anomalies ← QCD vectorlike

③ No $SU(N)\times U(1)^2$ anomalies $\mathrm{Tr}(T_{SU(N)})=0$

④ ~~…~~ Only $SU(3)^2 U(1)$ & $SU(2)^2\times U(1)$ possible

⑤ $U(1)_Y^3$ anomalies

~~…~~

$SU(3)^2$ yields

$$\mathrm{Tr}[T^a,[T^b,Y]_+] \sim \delta^{ab}\Big(\sum_L Y_\ell - \sum_R Y_r\Big)$$

only from quarks:

$$= 6Y_Q - 3Y_u - 3Y_d = 0\,!$$

$SU(2)^2$ yields

$$\mathrm{Tr}(\tau^a,[\tau^b,Y]_+) \sim \delta^{ab}\sum_L Y_\ell = 2Y_L + 6Y_Q = 0$$

$Y \sim$ hypercharge quantum # ($U(1)$ bit)

In SM both expressions yield zero!

(gravitational anomalies also cancel...)

Anomaly matching

Anomalies can sometimes yield valuable insight into non-perturbative physics

Consider QCD which has global symmetry group $SU(3)_L \times SU(3)_R \times U(1)_V = G$

(3 flavors ↗)

In principle there are G^3 anomalies but since these symmetries are not gauged they have no physical consequence

However, imagine weakly gauging them.

For consistent theory need to add "spectator" fermions to cancel off the anomaly

This assumes that for weak gauging this does not affect the physical spectrum of theory.

↑ all this discussion relates to theory in U.V.

Now what happens in I.R ?

't Hooft argued that the cancellation of anomalies must also hold there.

But in QCD we no longer have massless quarks in I.R - mesons / baryons instead

However, there is another massless state - the pion - a Goldstone boson

Can add to the low energy effective $\mathcal{L}$ a new term chosen to cancel the anomaly of spectator fermions in I.R

$$\mathcal{L} = N_c e^2/_{16\pi^2} \; \pi^0 \, \epsilon F F \qquad N_c = \# \text{ colors}$$

Now, run this argument around. Suppose chiral symmetry did not break (but confinement still occurs)

→ What massless states remain to cancel anomaly ?

massless composite states?

Ans

Need to make massless baryon out of

3 quarks

But $3 \otimes 3 \otimes 3 \to 10 \oplus 8 \oplus 8 \oplus 1$

real

so would be forced to

use 10 to get

non-zero $A(R)$

but $A(10) = 27$!

no way to cancel $A(\text{spectator}) = -3$

$\therefore$ QCD must break chiral symmetry if

it confines