Lecture 19

Spontaneous Symmetry Breaking

Consider scalar field

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \tfrac{1}{2}m^2\varphi^2 - \lambda/4!\,\varphi^4$$

Suppose $m^2 < 0$

write $V(\varphi) = \lambda/4!\,(\varphi^2 - v^2)^2 - \lambda/4!\,v^4$

$\uparrow$ drop

$v = \sqrt{\frac{6|m^2|}{\lambda}}$

2 classical configs minimize energy

$\varphi = \pm v$ related by original Z_2 symmetry $\varphi \to -\varphi$

in QM one expects unique ground state

lowest energy state corresponds to mix of 2 classical states via tunneling configs

But in QFT $\langle 0_+ | H | 0_- \rangle \to 0$ as $V \to \infty$

since must flip $\propto$ # dof

$\therefore$ 2 vacua remain degenerate $\langle 0_+ | 0_- \rangle = 0$

must pick either $|0_+\rangle$ or $|0_-\rangle$ as ground state

~~Pta~~ Ground state must break Z_2 symmetry

let $\rho = \varphi - v$

$$V(\varphi) = \frac{1}{6}\lambda \rho^2 v^2 + \frac{1}{6}\lambda v \rho^3 + \frac{1}{24}\lambda \varphi^4$$

ie $$\frac{\text{mass}^2}{2} = \frac{1}{6}\lambda v^2$$

action no longer respects $\varphi \to -\varphi$ symmetry

Might expect more Z's now needed to renormalize theory

In fact no! Effective action inherits Z_2 symmetry of classical theory. General result!

Allows for massive vector theories to be renormalizable if they result from spontaneous symmetry breaking like the S.M.

Continuous Symmetries

eg/ $\mathcal{L} = \partial_\mu \varphi \partial^\mu \varphi^\dagger - m^2 \varphi^\dagger \varphi - \frac{1}{4}\lambda (\varphi^\dagger \varphi)^2$

invariant under $U(1)$

$$\varphi \to e^{i\alpha} \varphi$$

if $m^2 < 0$ minima of V obtained when

$$\varphi = \frac{v}{\sqrt{2}} e^{-i\theta} \qquad v = [4m^2/\lambda]^{1/2}$$

θ - arbitrary

∞ family of ground state

Again pick 1 in QFT

$$\varphi = \frac{1}{\sqrt{2}}(v+\rho)\, e^{-\chi/v}$$

$$\Rightarrow \mathcal{L} = \frac{1}{2}\partial_\mu \rho \partial^\mu \rho - \frac{1}{2}(1+\rho/v)^2 \partial^\mu \chi \partial_\mu \chi$$

$$- |m^2| \rho^2 - \frac{1}{2}\sqrt{\lambda}\, |m| \rho^3 - \frac{1}{16}\lambda \rho^4$$

ρ mass

interactions

no mass term for χ!

Goldstone boson

General thm : when continuous global symmetry ④

breaks massless particles appear

correspond to flat directions in original

potential linking different vacua.

massless property survives to all orders

Why? all interactions derivatively

coupled

$\rho\chi\chi$ $\rho\rho\chi\chi$ contain factors of momentum

$\rightarrow$ vanish if $p \to 0$

broken $\mathcal{L}$ invariant under $\chi \to \chi + \alpha$ } which is not property of χ^2 term

Alternatively

$$\Gamma(\phi) = \Gamma(e^{i\alpha}\phi)$$

must be flat directions in Γ corresponds to phase of ϕ $\leftarrow$ Goldstones

Non-abelian case

eg/ $V(\phi) = \frac{\lambda}{4!}(\phi_i \phi_i - v^2)^2$

$i = 1 .. N$ SO(N) symmetry

choose $\phi_i = v \delta_{iN}$ say

Consider infinitesimal SO(N) trans acting on vacuum

$$v_i' = e^{i\theta \cdot T} v_i = v_i + i\theta^a T^a_{ij} v_j$$

subset of generators where

$T^a_{ij} v_j = 0$ remain unbroken

vev invariant under these symmetries

ie vacuum invariant

If $T^a v \neq 0$ generator is broken

for this case $T^a_{iN} \neq 0$ $\leftarrow (N-1)$

ie any generator with nonzero entry in last column

SO(N) → antisymmetric matrices element 1/−1

$$SO(N) \longrightarrow SO(N-1).$$

think of $SO(3) \to SO(2)$

$\phi_N = v+\rho \quad \phi_i \quad i=1\ldots N-1$ (6)

$\mathcal{L} =$

$$V(\phi) = \frac{\lambda}{4!}\left[(v+\rho)^2 + \phi_i\phi_i - v^2\right]^2$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i + \frac{1}{2}\partial_\mu\rho\,\partial^\mu\rho$$

$$-\frac{\lambda}{4!}\left(\rho^2+\phi_i\phi_i\right)^2 - \frac{\lambda}{4!}(2v\rho)^2 - \frac{\lambda}{4!}2\times v\rho(\rho^2+\phi_i\phi_i)$$

explicit $SO(N-1)$ symmetry

$(N-1)$ ϕ_i massless $\to$ GBs

$$\frac{1}{2}N(N-1) - \frac{1}{2}(N-1)(N-2)$$

$$= \frac{1}{2}(N-1)\left[N - N+2\right]$$

$$= \underline{N-1}$$

O.k what about gauged symmetries?

$$\mathcal{L} = -D_\mu \varphi D^\mu \varphi^\dagger - V(\varphi^\dagger \varphi) - F_{\mu\nu} F^{\mu\nu}.$$

abelian Higgs model.

assume $\langle \varphi(x) \rangle = v/\sqrt{2}$

write $\varphi(x) = \frac{1}{\sqrt{2}}(v+\rho)\, e^{-\chi/v}$.

potential only depends on ρ. χ is <u>massless</u> <u>would-be</u> GB - broken U(1) symmetry

<u>BUT</u> in gauge case can always make gauge transformation ~~that~~ shifts phase of $\varphi(x)$ by arbitrary function

$\Rightarrow$ choose $\chi = 0$ with this gauge freedom

"going to unitary gauge"

$$(D_\mu \varphi)^\dagger (D^\mu \varphi) \Rightarrow$$
$$\frac{1}{2}\left(\partial_\mu \rho + ig(v+\rho)A_\mu\right) \times$$
$$\left(\partial^\mu \rho - ig(v+\rho)A^\mu\right)$$

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$\rightarrow \frac{1}{2} \partial_\mu \rho \, \partial^\mu \rho + \frac{1}{2} g^2 (v+\rho)^2 A_\mu A^\mu$

$\begin{cases} M_{gauge} = gv \\ \text{no } X \text{ field.} \end{cases}$ — Higgs mechanism

alternatively keep GB X
& freeze ρ
$\hookrightarrow (igA_\mu - i\partial_\mu X/v) v e^{-iX/v} \times \text{c.c}$
gauge symmetry
$\delta A_\mu = \partial_\mu \alpha$
$\delta X = \alpha / gv$
pick $\partial A = 0$
$\hookrightarrow$ decouple A_μ, π

Non-abelian case

$$(D_\mu \phi)^\dagger (D^\mu \phi)$$

with $D_\mu \phi = \partial_\mu \phi_i - i g A_\mu^a T^a_{R\,ij} \phi_j$

$$\langle \phi_i \rangle = \frac{1}{\sqrt{2}} v_i$$

mass term for gauge field $\rightarrow$

$$\frac{1}{2} g^2 (A_\mu^a T^a_{R\,ij} v_j \; A^{\mu b} T^b_{R\,ik} v_k^*)$$

observe

① if $Tv = 0$ no mass term generated

② if $Tv \neq 0 \rightarrow$ massive gauge boson

breaking pattern depends on rep of scalars.

$$T^{a\,R}_{ij} v_j \neq 0 \quad \& \quad M^2_{ab} = \frac{g^2}{2} (T^a_R \cdot v)(T^b_R \cdot v)$$

es if φ is fundamental of $SU(N)$

can use gauge trans to make VEV in last component of φ (& real)

$\therefore$ Any generator with no element in last column remains unbroken

$\hookrightarrow$ form $SU(N-1)$ group

3 classes of broken generators

a) $T^a_{iN} = +1 \qquad i \neq N \qquad (N-1)$ of these

b) $T^a_{iN} = i \qquad i \neq N \qquad (N-1)$ of these

+ c) $T = \text{diag}(1 \ldots 1, -N-1)$

masses of a, b $= \frac{1}{2} g v$ & transform as fund of $SO(N-1)$

c) singlet in $SU(N-1)$ mass $= \left[\frac{(N-1)}{2N}\right]^{1/2} g v$.

Adjoint Breaking

Let $\phi = \phi^a T^a$ matrix valued

$$D_\mu \phi = \partial_\mu \phi - ig [T^a, \phi] A_\mu^a$$

$V = \langle \Phi \rangle =$ traceless hermitian matrix $(N \times N)$

$$\Rightarrow M^2 = \tfrac{1}{2} g^2 \, \mathrm{Tr}\left([T^a, V][T^b, V]\right)$$

use global $SU(N)$ to bring V to diagonal form

In general V will have N_1 eigenvalues v_1

N_2 eigenvalues v_2 etc

$$\sum N_i v_i = 0 \quad (\text{traceless } \Phi)$$

All generators which have non-zero entries that lie entirely within i^{th} block commute with V & form an unbroken $SU(N_i)$ subgroup

+ linear combination of generators & V itself $\to U(1)$

$$\Rightarrow SU(N_1) \times SU(N_2) \times \ldots U(1)$$

eg $SU(5)$

if $V = \left(-\frac{1}{3}\ -\frac{1}{3}\ -\frac{1}{3}\ \frac{1}{2}\ \frac{1}{2}\right)$

$$5\left\{\begin{pmatrix} \begin{pmatrix} \ & \ & \ \\ \ & \ & \ \\ \ & \ & \ \end{pmatrix} & \\ & \begin{pmatrix} \ & \ \\ \ & \ \end{pmatrix} \end{pmatrix}\right. \begin{array}{l} \leftarrow 3\times 3 \text{ block} \\ \\ \leftarrow 2\times 2 \text{ block} \end{array}$$

$$SU(5) \xrightarrow{\text{adj}} \underline{SU(3)\times SU(2)} \times U(1)$$

later . .