Recap

SSB

(1) Vacuum does not respect symmetry of $\mathcal{L}$

(2) Global symmetry $\rightarrow$ massless GB for each broken generator

(3) Gauge theory: gauge away GB but gauge fields acquire masses (broken)

(4) Renormalization of theory same as symmetric phase.

Today

Complete description non-abelian theory & discuss quantization of spontaneously broken gauge theory

(I) Adjoint breaking

Suppose $\Phi = \Phi^a T^a$ matrix valued $\ast$

& $D_\mu \Phi = \partial_\mu \Phi - ig [T^b, \Phi] A_\mu^b$

ie $\partial_\mu \varphi^a - i g f^{abc} A_\mu^b \varphi^c$

See equivalence by taking trace of $\ast$ with T^a

& $f_{abc} = \mathrm{Tr}\left(T^a [T^b, T^c]\right)$

Thus broken phase characterized by

$\langle \Phi \rangle$ non-zero

hermitian traceless matrix

$$\Longrightarrow \quad M^{2\,ab} = \frac{g^2}{2} \mathrm{Tr}\left[[T^a, V][T^b, V] \right]$$

use global SU(N) transformations to bring

M^2 to diagonal form

In general will then have

N_1 eigenvalues v_1

N_2 eigenvalues v_2

- - -

$\sum N_i v_i = 0 \leftarrow$ <u>traceless</u>

All generators whose non-zero entries lie wholly within ith block commute with V & form an unbroken $SU(N_i)$ subgroup

In addition there will always be a diagonal ~~...~~ generator $\propto V$ itself $\longrightarrow$ additional $U(1)$

eg $SU(5)$

& $V \sim \mathrm{diag}\left(-\frac{1}{3}\ -\frac{1}{3}\ -\frac{1}{3}\ \frac{1}{2}\ \frac{1}{2}\right)$

any T of form

$$\begin{pmatrix} \overbrace{\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}}^{3} \Big\}3 & \\ & 2\Big\{\overbrace{\begin{pmatrix} & \\ & \end{pmatrix}}^{2} \end{pmatrix}$$

commutes with V

3×3 block $\to$ generators of $SU(3)$ subgroup

$2\times 2 \to SU(2)$

thus $SU(5) \xrightarrow{\text{adj}} SU(3) \times SU(2) \times U(1)$

$\uparrow$

$T \propto V$ itself

later when discuss GUTs

Quantization of broken

SO(N) theory

let $\phi_i = v_i + \chi_i(x)$

$$\rightarrow (D_\mu \phi)_i = \partial_\mu \chi_i - ig A_\mu{}^a \underbrace{T^a_{ij}}_{\tau^a_{ij}} (v+\chi)_j$$

let $F^a_i = ig T^a_{ij} v_j$

$$\hookrightarrow (D_\mu \phi)_i = \partial_\mu \chi_i - A_\mu{}^a (F^a + \tau^a \chi)_i$$

$$\therefore (D_\mu \phi)_i (D^\mu \phi)_i =$$

$$(\partial_\mu \chi_i - A_\mu{}^a (F^a + \tau^a \chi)_i) \times$$

$$(\partial^\mu \chi_i - A^{\mu b} (F^b + \tau^b \chi)_i)$$

$$= (\partial_\mu \chi_i)(\partial^\mu \chi_i) + A_\mu{}^a A^{\mu b} (F^a + \tau^a \chi)_i (F^b + \tau^b \chi)_i$$

$$- A_\mu{}^a (F^a + \tau^a \chi)_i \partial^\mu \chi_i$$

$$- \partial_\mu \chi_i A^{\mu b} (F^b + \tau^b \chi)_i \leftarrow \text{A}$$

ie, $M^{2\,ab}_{\text{Gauge}} = F^a_i F^b_i = (FF^T)^{ab}$

≥ 0

every real rectangular matrix

F^a_i can be decomposed as

$$F^a_i = S^{ab} (M^b \delta^b_j) R_{ji}$$

diagonal matrix

vector boson masses

$$F F^T \sim S D^2 S^T$$

or $\tilde{A}_\mu^a = S^{ba} A_\mu^b$

to quantize need to impose gauge fixing term.

choose (R_ξ gauge)

$$G^a = \partial_\mu A^{\mu a} - \xi F^a_i X_i$$

$$\frac{1}{2\xi} G^{a2} \sim \frac{1}{2\xi} \Big[(\partial \cdot A)^2 + \xi^2 (F^a_i X_i)^2 - 2\xi \partial_\mu A^{\mu a} F^a_i X_i \Big]$$

integrate by parts

Cancels ⊗ piece

coupling kinetic term for X with A_μ.

Also yields mass op for X_i:

$$\sum F^a_i F^{*a}_j = \sum (F^T F^*)_{ij}$$

$\leftarrow$ diagonalize with R_{ij}

$\tilde{X} = RX$

ie masses just $\sqrt{\xi}\, M_V$ ... same

Ghost Lagrangian

$$\frac{\delta G^a}{\delta \theta^b} = -\partial^\mu D_\mu^{ab} + \xi F^a_j \, \tau^b_{jk} (v+X)_i$$

$$= -\partial^\mu D_\mu^{ab} + \xi F^a_j F^b_j$$

$$+ \xi F^a_j \, \tau^b_{jk} X_k$$

ie
$$\mathcal{L}_{gh} = -\partial^\mu \bar{c}^a D_\mu^{ab} c^b$$

$$- \xi (M^2)^{ab} \bar{c}^a c^b$$

ghosts now have mass too

$$- \xi F^a_j \, \tau^b_{jk} X_k \bar{c}^a c^b$$

$\uparrow$ new ghost-scalar interaction

Propagators

$$\langle A_\mu^a A_\nu^b \rangle \sim \frac{i}{p^2 - m_V^2}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2 - \xi m_V^2}(1-\xi)\right)\delta^{ab}$$

unphysical scalars (GB)

$$\langle \chi\chi \rangle \qquad \frac{i}{p^2 - \xi m_V^2}\,\delta^{ab}$$

ghosts:

$$\langle \bar{c} c \rangle \qquad \frac{i}{p^2 - \xi M_V^2}\,\delta^{ab}$$

notice: unitary gauge

$$\langle AA \rangle \sim \frac{\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{M_V^2}\right)}{p^2 - M_V^2}$$

$$\left.\begin{aligned} \langle \chi\chi \rangle &\sim 0 \\ \langle \bar{c} c \rangle &\sim 0 \end{aligned}\right\}$$ only physical states remain

BUT renormalizability difficult to prove since propagators fall off slowly with p

→ divergences in Feynman diagrams

easier for finite ξ when propagators $\sim \frac{1}{p^2}$

Spectrum in R_ξ gauge

N_u massless unbroken gauge fields

N_B massive vectors

$$(M_V^2)^{ab} = F^a_i F^b_i = (F F^T)^{ab}$$

with $F^a_i = i g T_{ij} v_j$

N_u, N_B determined by # zero eigenvalues of M^2

N_B massive (unphysical) scalars

$$\text{mass} \sim \sqrt{\xi}\,(M^2)_V$$

(would be GB as $g \to 0$)

N_a - physical scalars (Higgs)

mass given by non-zero eigenvalues of

$$\frac{\partial^2 V}{\partial \phi_i \partial \phi_j}$$

N_B massive ghosts

Mass matrix $\sqrt{\xi}\,(M^2)^{ab}$

physics indep of ξ $\quad \xi \to \infty$ unitary gauge