Standard Model

Example of chiral gauge theory – contains Weyl fermions in complex reps of gauge group.

[nevertheless it is anomaly free & hence consistent since $\sum_R T\{T,T\} = d_{abc} = 0$]

Consequence: contains only massless fermions (no gauge inv mass terms possible)

SM fermion masses obtained by coupling in gauge invariant manner Weyl fields to elementary scalar fields – Higgs

After SSB $\langle\phi\rangle \neq 0$ (fixed gauge).

$$\hookrightarrow y\bar{\psi}_L \psi_R \langle\phi\rangle$$

mass term

SSB also generates massive gauge fields → $W^\pm, Z^0$ bosons needed for weak interactions (short range)

Gauge group

$$SU(3) \times SU(2) \times U(1)$$

QCD → $SU(3)$: quarks come in 3 colors

$SU(2) \times U(1)$: EW group

↑ quarks/leptons come in 2 flavors

$U(1)$ – hypercharge – related to $U(1)_{EM}$

3 couplings: g_s, g_1, g_2

Representations:

$$\psi_L = \frac{1}{2}(1-\gamma_5)\psi$$

$Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ ← lefthanded quark doublet (2) of $SU(2)$

$L_L \equiv \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$ ← lepton doublet of $SU(2)$

Right handed fields are $SU(2)$ singlets u_R, d_R, e_R

↑ ν_R?

weak int^n maximally parity violating

Actually Any more complicated

3 families with this structure.

$$Q_L^i \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L \quad i=1,2,3.$$

$$\mathcal{L} = \bar{Q}_L^i \not{D} Q_L + \bar{L}_L^i \not{D}' L_L^i$$
$$+ \bar{u}_R^i \not{D}'' u_R^i + \bar{d}_R^i \not{D}''' d_R^i$$
$$+ \bar{e}_R^i \tilde{\not{D}} e_R + \mathcal{L}_{Higgs} + \mathcal{L}_{Yuk}$$
$$+ \mathcal{L}_{gauge}$$

$\not{D}, \not{D}'$ etc different covariant derivs either diff reps of SU(2) / SU(3) or different hypercharges

controls couplings to U(1)

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questions:

why 3 families?

why strange P-violating reps?

Is there ν_R?

What sets up Yukawa couplings

$$m_t / m_\nu \sim y_t / y_\nu = 10^{12}$$

Gauge / Higgs sector

Higgs must also be doublet of $SU(2)$
(for $\mathcal{L}_{Yuk}$ to be G.I.)

$$(D_\mu \phi)_i = \partial_\mu \phi_i - i\left(g_2 A_\mu^a T^a + g_1 B_\mu Y\right)_i^{\,j} \phi_j$$

with $T^a = \frac{1}{2}\sigma^a$

$Y = -\frac{1}{2} I$ ← later...

Matrix form ⟹

$$\frac{1}{2}\begin{pmatrix} g_2 A_\mu^3 - g_1 B_\mu & g_2(A_\mu^1 - iA_\mu^2) \\ g_2(A_\mu^1 + iA_\mu^2) & -g_2 A_\mu^3 - g_1 B_\mu \end{pmatrix}$$

assume $V(\phi) \sim \lambda/4 \, (\phi^\dagger \phi - v^2)^2$

take $\langle \phi \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}$

use SU(2) transformations to put vev into this form

As before gauge field masses determined by (5)

$$\mathcal{L}_{mass} = -\frac{1}{8} v^2 g_2^2 (1,0) \cdot \begin{pmatrix} A_\mu^3 - \frac{g_1}{g_2} B_\mu & \sqrt{2} W_\mu^- \\ \sqrt{2} W_\mu^+ & -A_\mu^3 - \frac{g_1}{g_2} B_\mu \end{pmatrix}^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} (A_\mu^1 \pm i A_\mu^2) = W_\mu^\pm$$

$$\text{let } \tan\theta_W = \frac{g_1}{g_2}$$

$$\therefore A_\mu^3 - \tan\theta_W B_\mu = \frac{1}{\cos\theta_W} \underbrace{(\cos\theta_W A_\mu^3 - \sin\theta_W B_\mu)}_{Z_\mu}$$

$$\therefore \mathcal{L}_{mass} = \left(\frac{g_2 v}{2}\right)^2 W^{+\mu} W_{-\mu} - \frac{1}{2}\left(\frac{g_2 v}{2 c_W}\right)^2 Z^\mu Z_\mu$$

thus
$$\left.\begin{aligned} M_W &= g_2 v/2 \\ M_Z &= M_W/\cos\theta_W \end{aligned}\right\}$$

$$\sin^2\theta_W \,(\text{expt}) = 0.223$$

Note orthogonal combination

$$A_\mu = \sin\theta_W A_\mu^3 + \cos\theta B_\mu$$

remains massless ← photon.

(One of) original covariant derivatives

is $D_\mu = \partial_\mu - i g_2 A_\mu^3$

$= \partial_\mu - i g_2 (\sin\theta_W A_\mu + \cos\theta_W Z_\mu)$

↑ photon

thus $e = g_2 \sin\theta_W$ (G_F will also involve g_2, θ_W)

Hyperchage

Need to assign hypercharges to quarks & leptons. This will be determined by couplings to A_μ^3, B.

$g_2 A_\mu^3 T^3 + g_1 B_\mu Y$

$$= \frac{e}{\sin\theta_W}(\sin\theta_W A_\mu + \cos\theta_W Z_\mu) T^3$$

$$+ \frac{e}{\cos\theta_W}(\cos\theta_W A_\mu - \sin\theta_W Z_\mu) Y$$

Thus/

$$e(A_\mu + \cot\theta_W Z_\mu)T^3$$
$$+ e(A_\mu - \tan\theta_W Z_\mu)Y$$
$$= e(T^3+Y)A_\mu + e(\cot\theta_W T^3 - \tan\theta_W Y)Z_\mu$$

Thus charge is given by

$$\boxed{Q = T^3 + Y}$$

← note: this is consistent with Higgs assignment of $Y = -1/2$ previously.

Since $t^3 \nu_L = \frac{1}{2}\nu_L$

$t^3 e_L = -\frac{1}{2} e_L \quad t^3 e_R = 0$ (singlet)

$\rightarrow Y\nu_L = -\frac{1}{2}\nu_L$, $Ye_L = -\frac{1}{2}e_L$, $Ye_R = -1 e_R$.

ie $Y L_L = -\frac{1}{2}$

neutral Higgs if $Y(H) = -\frac{1}{2}$

quarks similarly using $Q = t^3 + Y$

$$Y Q_L = \frac{1}{6} \quad \left(\frac{1}{2} + \frac{1}{6} = \frac{2}{3} \text{ up quark etc}\right)$$

$$Y u_R = \frac{2}{3} \quad Y d_R = -\frac{1}{3} \quad (Y\nu_R = 0)$$

Fermion Masses

Yukawa int^ns with Higgs

← leptons first

$$\mathcal{L}_Y = -y \, L_{Li} \, \bar{e}_R \, \phi_j \, \epsilon^{ij}$$

possible term

SU(2) invariant.

Lorentz invariant (L & R hel. coupled)

& hypercharge singlet if $Y L_L = -\frac{1}{2} L_L$ as we have assigned

& $Y \bar{e}_R = +1$.

in unitary gauge after SSB:

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} v+H \\ 0 \end{pmatrix} \qquad l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$\frac{-y}{\sqrt{2}} v \, e_L \bar{e}_R$$ Dirac mass term

with $m_e = yv/\sqrt{2}$

notice if no $\bar{\nu}_R$ cannot make Dirac neutrino mass

Quark sector

Consider

$$\mathcal{L}^{Q}_{Yuk} = -y' \, q_{L i} \bar{d}_{R} \phi_j \epsilon^{ij}$$

like lepton sector term

$+ h.c$

$$+ \quad -y'' \phi^{i\dagger} q_{L i} \bar{u}_R .$$

neglect SU(3) indices ...

- Lorentz invariant
- Gauge invariant
- hypercharge singlets

remember

$Y q_L = \frac{1}{6}$

$Y u_R = \frac{2}{3}$

$Y d_R = -\frac{1}{3}$

unitary gauge $\quad \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H \\ 0 \end{pmatrix}$

→ Dirac mass terms:

$$-\frac{y'}{\sqrt{2}} v \, \bar{d}_R d_L + h.c$$

$$-\frac{y''}{\sqrt{2}} v \, \bar{u}_R u_L + h.c.$$