Lecture 27

O.k ~~to~~ lets now generalize to include 3 families.

First the quarks.

$$\mathcal{L}_Y = - q_{L_i}^{I} \Gamma^{IJ} \bar{d}_R^{\,J} \epsilon^{ij} \phi_j$$

$$- \phi^{+i I} q_L \Gamma'^{IJ} \bar{u}_R^{\,J}.$$

where Γ, Γ' complex 3×3 matrices.

use unitary transformations to diagonalize almost

Γ, Γ' ← kinetic terms unchanged

$d \to D d \qquad \bar{d} \to \bar{D} \bar{d}$ ← (unchanged) later.

$u \to U u \qquad \bar{u} \to \bar{U} \bar{u}$

↳ $\Gamma \to D^T \Gamma \bar{D}$ ← down q masses (real, pos)

$\Gamma' \to U^T \Gamma' \bar{U}$ ← up q masses... (real, pos)

Diagonal kinetic terms (11)
undergo under these unitary transformations

Old diagonal terms look like

$$\bar{u}_L \gamma_\mu W^{\mu +} d_L$$

$$\hookrightarrow \bar{u}_L^I V^{IJ} \gamma_\mu W^{\mu +} d_L^J$$

where $V^{IJ} = (U+D)^{IJ}$

CKM matrix

unitary

9 real parameters 3 angle, 6 phases

but can still make phase rotations

$$D_I \to e^{i\alpha_I} D_I \,, \quad U_I \to e^{i\beta_I} U_I$$

& 5 invariances

Lead kinetic & mass terms invariant

$\longrightarrow$ just 4 parameters left

3 angles + 1 phase $\Rightarrow$ violates CP, T ...

$\Longrightarrow$

$$V = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}$$

θ_1 – Cabibbo angle

$s_1 = 0.224$
$s_2 = 0.041$
$s_3 = 0.016$
$\delta = 40^\circ$
} exactly.

presence of δ implies charged currents not time reversal invariant

$\longrightarrow$ CP-violation. ← T-violation if complex
↳ CP violation
If CPT exact

Back to lepton sector + 3 families

$$\mathcal{L}_Y = -y^{IJ} l_{Li}^{I} \bar{e}_k^{J} \phi_i \epsilon^{ij} + h.c$$

y^{IJ} complex 3×3 matrix

again, can make unitary transformations to diagonalize Yukawas

$$l_L \to L l_L \qquad \bar{e}_R \to \bar{E} \bar{e}_R$$

$$y \to L^T y \bar{E}$$

↖ masses $\frac{v}{\sqrt{2}}$ × eigenvalues (real, pos)

all ν_I massless SM

→ now know this to be incorrect $m_\nu \sim 10^{-2}$ eV or so

add

$$-\tilde{y} \phi^\dagger l_L \bar{\nu}_R$$

(single family)

SSB → $-\tilde{y} \frac{v}{\sqrt{2}} \nu_L \bar{\nu}_R$

"sterile" → $Q\nu_R = Y\nu_R = 0$ gauge singlet

$R = \bar{\nu}_R \phi \nu_R$

Since ν_R sterile another mass term possible

$$M\, \bar{\nu}_R^T C \bar{\nu}_R + h.c$$

Lorentz & gauge invariant

term is forbidden if lepton # conserved

ex $\nu_R \to e^{i\alpha}\nu_R$ etc.

total mass matrix :

$$(\nu_L \; C\bar{\nu}_R)\, C \begin{pmatrix} 0 & \tilde{m} \\ \tilde{m} & M \end{pmatrix} \begin{pmatrix} \nu_L \\ C\bar{\nu}_R \end{pmatrix}$$

where $\tilde{m} = y v/\sqrt{2}$

if $M \gg \tilde{m}$ (possible since M U.V scale & protected by lepton # in addition conserved)

$\longrightarrow$ eigenvalues:

$$M_1 \; \tilde{m}^2/M \ll M$$

See saw mechanism $\leftarrow$ ν_L light
ν_R heavy
natural

3 families...

$$\mathcal{L}_Y = -\epsilon^{ij}\phi_j^\dagger \, l_i^I \, y^{IJ} \bar{e}_R^J$$

$$- \varphi^{\dagger i} l_i^I y'^{IJ} \bar{\nu}_J$$

$$- \frac{1}{2} M^{IJ} \bar{\nu}_I^c \nu_J + h.c$$

M, y, y' complex 3×3 matrices

Unitary gauge:

$$\mathcal{L} \to -\frac{v}{\sqrt{2}} e_{L}^{I} y^{IJ} \bar{e}_R^J - \frac{v}{\sqrt{2}} \nu_L^I y'^{IJ} \bar{\nu}_R^J$$

$$- \frac{1}{2} M^{IJ} \bar{\nu}_R^I \nu_R^{cJ} + h.c$$

Integrate out $\bar{\nu}_I$ fields by completing square $\Longrightarrow$

$$(M_\nu)_{IJ} = -\frac{v^2}{2} \left(y'^T M^{-1} y'\right)_{IJ}$$

mass matrix for ν_L states (light guys)

As before can make unitary transformation

$$e \to E e \quad \bar{e} \to \bar{E}\bar{e} \quad \nu \to N\nu$$

diagonal parts of kinetic terms unchanged

matrices y, m_L replaced by

$$E^T y \bar{E}, \quad N^T m_L N$$

As for the quarks the charged (off diagonal) current transforms like

$$\bar{\nu}_L N^\dagger E \not{W}^+ e_L$$

PMNS matrix $\to$ X = analog of CKM matrix.

6 parameters (not 4) (neutrino Majorana mass term means not possible to do phase transformations on ν fields)

$\Rightarrow$ neutrino oscillations

PMNS matrix

Neutrino Oscillations

Neutrinos produced by scattering of electrons on some target are

linear combination of mass eigenstates

$$\nu_I' = X_{IJ}\,\nu_J$$

Different mass eigenstates propagate at different speeds ⇒ hence ν_e may transform into ν_μ, ν_τ as propagate away from source

⟶ seen by production of muon/tau in detector.

In fact this is the primary evidence for neutrino mass ✱✱

no hierarchy in θ angles for neutrino sector ??