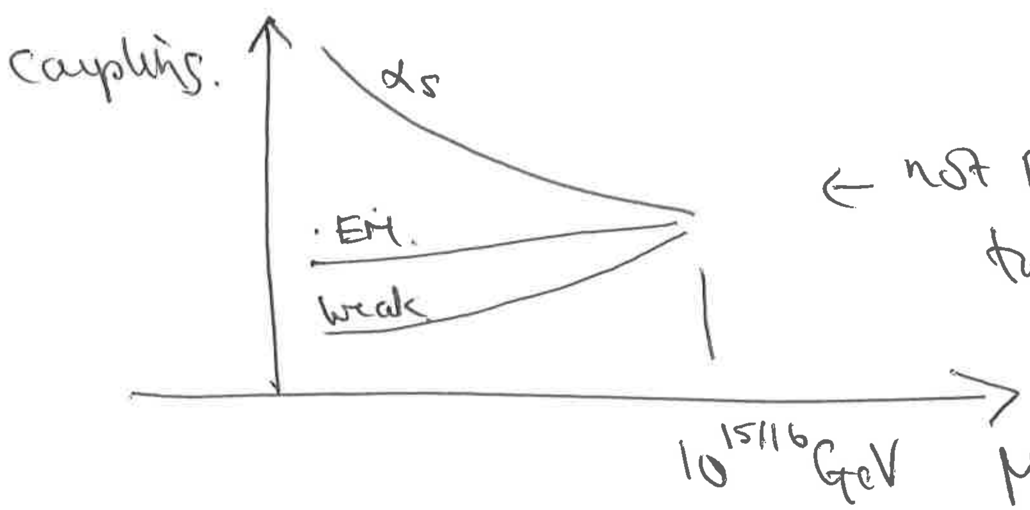
Grand Unified Theories GUT

SM contains lots of parameters (~20)

Fermions assigned in complicated reps of direct product of symmetry groups with 3 independent couplings



← not perfect but tantalising.

Hint: perhaps SM results from SSB of larger simple gauge group at $E \gg E_{EW}$?

Hope is that such a model will have fewer parameters & more prediction power...

Consider simplest such model — SU(5)

Georgi-Glashow

Discussed earlier

Can break $SU(5) \to SU(3) \times SU(2) \times U(1)$ by adding scalar in adj rep of $SU(5)$

$$\langle \phi \rangle = \begin{pmatrix} \left.\begin{pmatrix} -\frac{1}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3} \end{pmatrix}\right\} 3 & \\ & 2 \left\{ \begin{pmatrix} \frac{1}{2} & \\ & \frac{1}{2} \end{pmatrix} \right. \end{pmatrix}$$

Can be achieved with potential of form

$$V(\phi) \sim \mathrm{Tr}(\phi^{\dagger}\phi) + \mu\, \mathrm{Tr}(\phi^{\dagger}\phi)^2 + \lambda \left[\mathrm{Tr}(\phi^{\dagger}\phi)\right]^2$$

$SU(5)$ 24 generators

12 survive in SM $(8+3+1)$.

$\therefore$ 12 broken generators $\to$ 12 massive gauge bosons. Since reps of $SU(5)$ will contain <u>both</u> leptons / quarks $\to$ mediate proton decay

M_X <u>must</u> be large.

Biggest question - how to assign fermions to reps of SU(5) ?

3

SM fermions :

$$u_L^i \quad i=1,2,3 \text{ color } + d_L^i$$

$$d_R^i, \; u_R^i \qquad e_L, \nu_L, e_R, (\nu_R ?)$$

labeling by SM quantum #s

$$q_L = \left(3, 2, \tfrac{1}{6}\right)_L \; u_R = \left(3, 1, \tfrac{2}{3}\right)_R \; d_R = \left(3, 1, -\tfrac{1}{3}\right)$$

$$l_L = \left(1, 2, \tfrac{-1}{2}\right)_L \; e_R = (1, 1, -1)_R \; [\bar{\nu}_R = (1,1,0)]$$

Since $\bar{u}_R$ ~~...~~$_L$ transforms like $(u^c)_L$

can write everything in terms of lefthanded fields by

$$\left(3,2,\tfrac{1}{6}\right) \quad \underset{\bar{u}_R}{\left(\bar{3}, 1, -\tfrac{2}{3}\right)} \quad \bar{d}_R \left(\bar{3}, 1, \tfrac{1}{3}\right)$$

$$\left(1, 2, -\tfrac{1}{2}\right) + \underset{\bar{e}_R}{(1, 1, 1)} \qquad [\bar{\nu}_R = (1,1,0)]$$

O.K. how does fundamental of $SU(5)$ break down under $SU(3) \times SU(2) \times U(1)$

if ψ^μ is fundamental rep

$\underbrace{\mu = 1,2,3}_{\alpha} \sim SU(3)$ $\underbrace{\mu = 4,5}_{i}$ $SU(2)$ label

α under $SU(3)$

$\psi^\alpha \sim 3$ & has hyperharge $-\frac{1}{3}$ & singlet under $SU(2)$

$\psi^i \sim 2$ under $SU(2)$, singlet under $SU(3)$

& hyperharge $\frac{1}{2}$

ie

$$5 \to (3, 1, -\tfrac{1}{3}) \oplus (1, 2, \tfrac{1}{2})$$

<u>or</u> $$\bar{5} \to (\bar{3}, 1, \tfrac{1}{3}) \oplus (1, 2, -\tfrac{1}{2})$$

Here are 2 of the SM reps we need!

still have 10 SM fields to fit in (<u>excluding</u> $\bar{\nu}_R$)

one candidate $SU(5)$ rep is antisymmetric $\underline{10}$

$\psi^{\mu\nu}$.

Since 10 is antisymmetric product of 2 5's we just need to figure out

$$\left[(3,1,-\tfrac{1}{3}) \oplus (1,2,\tfrac{1}{2}) \right] \otimes \left[(3,1,-\tfrac{1}{3}) \oplus (1,2,\tfrac{1}{2}) \right]$$

& take antisymmetric part

Now

$$(3,1,-\tfrac{1}{3}) \otimes_A (3,1,-\tfrac{1}{3}) = (\bar{3},1,-\tfrac{2}{3})$$

[$3 \times 3 = 9$ states, 3 in antisymm $\psi^\mu \psi^\nu \epsilon_{\mu\nu\lambda} \to \psi_\lambda = \bar{\psi}^\lambda$]

and

$$(3,1,-\tfrac{1}{3}) \otimes_A (1,2,\tfrac{1}{2}) = (3,2,\tfrac{1}{6})$$

finally

$$(1,2,\tfrac{1}{2}) \otimes_A (1,2,\tfrac{1}{2}) = (1,1,1)$$

$$2 \otimes 2 = \psi^\alpha \psi^\beta \epsilon_{\alpha\beta} = \text{singlet } 1$$

SM reps exactly fit in $10 + \bar{5}$!
(Remarkable) !!

Explicitly

$$\bar{\psi}^i = (\bar{d}^r, \bar{d}^g, \bar{d}^b, \nu, e)$$

$$\chi^{ij} = \begin{pmatrix} 0 & \bar{u}^r & \bar{u}^g & d^r & u^r \\ -\bar{u}^r & 0 & \bar{u}^b & d^g & u^g \\ -\bar{u}^g & -\bar{u}^b & 0 & d^b & u^b \\ -d^r & -d^g & -d^b & 0 & \bar{e} \\ -u^r & -u^g & -u^b & -\bar{e} & 0 \end{pmatrix}$$

What does this buy us?

* electric charge related to generator of simple group ($SU(5)$). But spectrum of such group is quantized (recall angular momentum QM)

$\therefore$ electric charge is quantized

(no explanation for this in SM where comes from $U(1)$ factor)

Furthermore $\quad \mathrm{Tr}\, Q = 0 \quad$ (generator $SU(5)$)

$\bar{5}$ rep $\Rightarrow 3Q_{\bar{d}} = -Q_e$

ie $Q_d = -1/3$ electric charge!

* SU(5) predicts $\sin\theta_W$

[only 1 gauge coupling g_5]

g_1/g_2 determined by normalization of Y, to T_3.

$$\mathrm{Tr}\, T_3^2 = \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2 \text{ on 5 rep.}$$
$$= \tfrac{1}{2}$$

$(T_3\, \bar{d} = 0)$

$$\mathrm{Tr}\, Y^2 = 3\left(\tfrac{1}{3}\right)^2 + 2\left(\tfrac{1}{2}\right)^2 = \tfrac{5}{6}$$

$\therefore$ T_3 & $\sqrt{\tfrac{3}{5}}\,Y$ have equal normalization

$$\text{u } \tan\theta_W = \sqrt{\tfrac{3}{5}} = \frac{g_1}{g_2}$$

ball park right!

(need to RG evolve $M_X \to M_{EW}$ ..)

* SU(5) anomaly free $\therefore$ SM also

$d_{abc} \propto \mathrm{Tr}\, T^3$ where $T = \begin{pmatrix} 2 & & & & 0 \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ 0 & & & & -3 \end{pmatrix}$

$\mathrm{Tr}(T^3)_{\bar{5}} = 3(-2)^3 + 2(3)^3 = 30$ for $U(1)$

$\mathrm{Tr}(T^3)_{10} = 3(4)^3 + 6(-1)^3 + (-6) = -30$ total $= 0$!!

use charge operator rather than hypercharge

$$\frac{D(\bar{5})}{D(10)} = \frac{\mathrm{Tr}\, Q^3(\bar{5})}{\mathrm{Tr}\, Q^3(10)}$$

$$= \frac{3\left(\frac{1}{3}\right)^3 + (-1)^3 + 0^3}{3\left(-\frac{1}{3}\right)^3 + 1^3} = \frac{-1+\frac{1}{9}}{1-\frac{1}{9}}$$

$$= -1$$

$$\therefore\ D(\bar{5}) + D(10) = 0$$

$$\left[\begin{array}{l} \bar{5} \sim (\bar{d}, e, \nu) \\ 10 \sim (u, \bar{u}, d, \bar{e}) \end{array}\right]$$

Problems

* X bosons mediate proton decay
need large M_X to suppress. Incompatible with
gauge coupling unification ...

* Higgs fields embedded in $\bar{5}$ of $SU(5)$

$$(\phi^r \phi^b \phi^g \underbrace{\bar{\phi}, \phi^0}_{\text{SM model}})$$

new colored Higgs scalars ... ← not seen

* ν_R doesn't fit (singlet of $SU(5)$)

* Possible $SU(5)$ inv Yukawas for
fermion masses

$$-y H^i \psi^j X_{ij} - \frac{y'}{8} \epsilon^{ijklm} H_i^+ X_{jk} X_{lm}$$

$+ h.c$

↳ rewrite in SM fields
constrains structure of SM Yukawas ✓

but $\quad m_b = m_\tau \quad m_s = m_\mu \quad m_d = m_e$ ✗

* Higgs potential not stable to radiative corrections -- naturalness problem (gauge hierarchy problem)

$$M_H^2 \to M_X$$

fine tuning ...

* No explanation of chiral structure -- or family replication

<u>Improvements / extensions</u>

— Pati - Salam ($SU(4) \times SU(2)_L \times SU(2)_R$)

— <u>SO(10)</u>

$\Rightarrow$ all SM fermions <u>including</u> ν_R fit into <u>single</u> (16) rep of SO(10).

<u>Good</u> ✓.

But again difficulties accommodating fermion masses ..

Patti Salam

LR asymmetry of SM puzzling → perhaps should be restored at high energies?

$$SU_L(2) \times SU_R(2)$$

with $SU_R(2)$ broken

Also quarks/leptons look similar from perspective of EW interactions. Could leptons be 4th color of quark?

$$SU(4) \times SU_L(2) \times SU_R(2) \quad ?$$

L handed fields

$$(4, 2, 1)$$

R handed fields

$$(\bar{4}, 1, 2)$$

If Higgs $(4,1,2)$ gets vev

↳ $(4,2,1) \to (3,2)_{\frac{1}{6}} \oplus (1,2)_{-\frac{1}{2}}$

$$(\bar{4},1,2) \to (\bar{3},1)_{\frac{1}{3}} \oplus (\bar{3},1)_{-\frac{2}{3}} + (1,1) + (1,1)_0$$

etc.

SO(10)

rank 5 group
contains SU(5) (& hence SM)
fundamental of SO(10) → $5 + \bar{5}$ under SU(5)

Perhaps more remarkably
16 (spinor) rep of SO(10)
→ $\bar{5} + 10 + 1$ under SU(5)

ie all fermions in one generation of SM
can be put into single rep of SO(10)!
SO(10) being real clearly has no anomaly...
* can under certain assumptions undo
proton decay problems
* now implies existence of ν_R ← ✓.

SU(5)×U(1) & SU(2)×SU(2)×SU(4)
maximal subgroups of SO(10)